

Exotic Branes and Symmetries of String Theory

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Abstract

Are duality transformations symmetries of string theory? For AdS space-time the answer is no for generic asymptotic values of the moduli, since the duality symmetry is broken explicitly in the dual conformal field theory. In contrast, in string theory in flat space-time, monodromies around codimension two exotic branes show that duality transformations are spontaneously broken discrete gauge symmetries with observable consequences, provided macroscopic loops of these branes are not hidden behind an event horizon. We discuss how this can be achieved and how the situation in flat space-time differs from that in AdS space-time. We also discuss observability of codimension two non-BPS branes, codimension one BPS and non-BPS branes and higher codimension branes of infinite tension.

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1 Introduction

String theory contains branes of different dimensions. Since these are extended objects, they have infinite energy and are not regular states of the theory. However one can construct closed loops of these branes by taking the spatial part of the world-volume of a p -brane to lie along a sphere S^p or any other compact p -dimensional subspace and get a finite energy state of the theory. Furthermore, as long as the size of S^p is large, this configuration will locally look like a flat p -brane and an asymptotic observer may be able to study its properties. This however requires that the configuration is not hidden from the asymptotic observer by the event horizon of a black hole.

For branes of codimension larger than two, there is no problem in producing a macroscopic loop that is not shielded by an event horizon. If we use Planck units to express all quantities, then for a codimension k brane of tension $\mathcal{T}$ in D space-time dimensions, a loop of size L will have mass $M \sim \mathcal{T} L^{D-k-1}$ and hence its Schwarzschild radius r_s will be of order $M^{1/(D-3)} \sim \mathcal{T}^{1/(D-3)} L^{(D-k-1)/(D-3)}$. For $k > 2$ this growth is slower than L for large L and hence for sufficiently large L the brane is always outside its would be Schwarzschild radius. For codimension two branes we get $r_s \sim \mathcal{T}^{1/(D-3)} L$ and hence whether or not this is larger than L depends on the tension of the brane.¹ Finally for codimension one brane we have $r_s \sim \mathcal{T}^{1/(D-3)} L^{(D-2)/(D-3)}$ and for large L this always exceeds the size L of the system. Thus in this case an asymptotic observer cannot study a large loop of such branes.

This shows that the codimension two branes are somewhat special since our ability to study them depends on the tension of the brane. String theory contains many codimension two branes, including exotic branes with the property that as we transport a state around such an exotic brane, the state undergoes a non-trivial monodromy transformation associated with some duality symmetry [2–10]. The existence of these branes can be used to argue that we must identify field configurations related by these duality transformations and hence the corresponding symmetry is a gauge symmetry. On the other hand at weak coupling, the tension of most exotic branes become large in the Planck scale and hence they are shielded by an event horizon and become inaccessible to an asymptotic observer. Therefore the question arises as to in what sense these branes exist and how we can argue that the monodromies associated with these branes are gauge symmetries.

For BPS exotic branes we suggest a possible way out of this. While they may be behind the event horizon at weak coupling, there is some corner of the moduli space where their tension becomes small in the Planck units, since typically they are related to fundamental strings, D-branes or other known objects via various duality symmetries, and the latter do become light in some corner of the moduli space. Now it was argued in [11–13] that irrespective of the asymptotic values of the moduli, string theory contains states for which in an arbitrary large region in space-time the moduli take any desired value, and furthermore, this region is not hidden from the asymptotic observer by an event horizon, thus avoiding the issues discussed in [14]. So one could in principle have a state in which in a large region of space-time the

¹The codimension two branes we consider here have logarithmically varying scalar fields away from the brane. So one might expect that the energy of a codimension two brane loop may have an additional factor of $\log L$ from the scalar field energy density and spoil the scaling argument given here [1]. We shall discuss in section 5 why this does not happen for the branes that we shall study.

exotic brane is light and is not hidden by an event horizon. Since this region is accessible to an asymptotic observer, such an observer can study the exotic brane and measure the monodromy associated with these branes, thereby establishing that these symmetries are indeed gauged. We discuss this for various types of codimension two branes in sections 2, 3 and 4, and argue that for all exotic BPS branes we can find appropriate regions in the moduli space where their tension becomes small in Planck units, and hence these brane loops can be studied by creating a region in space-time where the moduli take the desired values.

In section 5 we address the problem described in footnote 1, – the possibility of additional logarithmic factors in the energy of codimension two brane loops due to logarithmically varying scalar fields produced by the brane. We show that the scaling argument, that shows that the energy of the brane loop of size L should grow as L^{D-3} , relies on the hidden assumption that the energy of the brane does not depend on the microscopic cut-off given by the thickness of the brane. We then show how the variation of the dilaton removes the dependence on the microscopic cut-off for the BPS branes of the type analyzed in sections 2, 3 and 4, and reproduces the result based on the scaling argument.

As already mentioned, monodromies around codimension two branes form part of discrete gauge symmetries of the theory. We discuss this in detail in section 6 and use this to formulate a precise test of discrete gauge symmetries in string theory. This extends to the cases when the discrete symmetry is spontaneously broken, including the U-duality symmetries of the theory, all of which are spontaneously broken at a generic point in the moduli space of the theory.

The last point may seem somewhat surprising, since it was argued in [15, 16] that in anti de Sitter space-time, spontaneously broken gauge symmetries have no observable signature. Indeed type IIB string theory on $AdS_5 \times S^5$ has $SL(2, \mathbb{Z})$ S-duality symmetry that can be identified as the S-duality symmetry of the holographically dual $N=4$ supersymmetric Yang-Mills theory. However, for generic values of the complex coupling, the S-duality symmetry of the Yang-Mills theory is explicitly broken and has no direct observational signature, except possibly some BPS states whose existence is protected by supersymmetry [17]. In particular the status of S-duality is on the same footing as the $\phi \rightarrow -\phi$ symmetry in a scalar field theory with $\lambda\phi^3$ interaction. The transformation relates theories at λ and $-\lambda$ but is not a symmetry unless λ vanishes. We discuss in section 7 why the situation in AdS is different from that in flat space-time where we do have observable signature of the duality symmetry even for generic values of the asymptotic moduli.

For non-BPS exotic branes, some of which have been conjectured to exist in string theory,

the situation is more complicated, since we do not have *a priori* knowledge of their tension at any point in the moduli space. We discuss this in section 8.

In sections 9 and 10 we take a different approach to studying low codimension branes. Instead of studying them via finite energy configurations in the theory of interest, we start with a different string theory, and then realize both the desired string theory and the low codimension branes in this theory as a configuration in the parent string theory.

In section 9 we exploit the fact that in many string compactifications, various low codimension branes are already present as part of compactification. These branes extend along the non-compact part of the space-time but are localized along one or more compact internal directions. One class of examples of this type is F-theory compactifications where (p, q) seven branes, localized along two of the compact directions and extended along other space-time directions are present in the system [18–20]. Another example is provided by $E_8 \times E_8$ heterotic string theory whose strong coupling limit is M-theory ‘compactified’ on a line segment with end of the world 9-branes at the two ends of the segment, each carrying one set of E_8 gauge fields [21]. A third example involves type I’ theory [22] where we have type IIA orientifold 8-planes sitting at the two ends of an interval, with D8-branes parallel to the orientifold planes placed at various points along the interval and the regions between the D8-branes and orientifold planes are described by massive type IIA supergravity discovered by Romans [23]. By creating large regions in space-time where the size of the compact directions transverse and tangential to the branes becomes large, we can create a configuration where we have isolated flat branes that can be studied by an asymptotic observer.

In section 10 we consider a hypothetical flat codimension one brane, carrying positive energy density, that separates two Minkowski vacua A and B of string theory. Since the brane has infinite energy, it is not a state in theory A or theory B, but it is an allowed background in string theory as long as such a brane exists. We then explore to what extent an observer in A can explore the properties of B and vice versa. We find that even in this case, an observer remaining in A cannot study the properties of B to arbitrary precision, since there is a limit to how long an apparatus can remain in B in an inertial frame and still send back signal to the observer in A. It is however possible for an observer to spend infinite amount of time in A, finish all the desired measurements and then cross over to B and spend infinite amount of time there, performing all the desired measurements in B. Such an observer will have complete information on the observables of A and B,² but the price we pay is that the local environment

²In this paper, by observables of a theory we shall mean all the S-matrix elements of that theory.

of the incoming observer will differ from that of an outgoing observer.

2 $\text{SL}(2, \mathbb{Z})$ multiplet of strings in four dimensions

We begin our discussion with the $\text{SL}(2, \mathbb{Z})$ family of strings discussed in [2]. Heterotic string theory on $T^6 \times \mathbb{R}^{3,1}$ is invariant under an $\text{SL}(2, \mathbb{Z})$ symmetry:

$$\tau \rightarrow \tau' = \frac{p\tau + q}{r\tau + s}, \quad \begin{pmatrix} p & q \\ r & s \end{pmatrix} \in \text{SL}(2, \mathbb{Z}), \quad (2.1)$$

where

$$\tau = a_0 + i g_4^{-2} \equiv \tau_1 + i\tau_2, \quad (2.2)$$

Here g_4 is a suitably normalized four dimensional string coupling, related to the asymptotic value of e^ϕ where ϕ is the dilaton and a_0 is the asymptotic value of the axion, obtained by dualizing the 2-form field $B_{\mu\nu}$. The fundamental string, which we shall call the (1,0) string, has tension $1/(2\pi\ell_s^2)$ where $\ell_s = \sqrt{\alpha'}$ is the string length scale. Since in D space-time dimensions the Planck length ℓ_p is proportional to $\ell_s g_D^{2/(D-2)}$, g_D being the D -dimensional string coupling, we have $\ell_p \propto \ell_s g_4$ in $D = 4$. Therefore, if we work in units where the Planck length ℓ_p is set equal to one, then the tension $T_{1,0}$ of the fundamental string would be given by,

$$T_{1,0} = C g_4^2 = C \tau_2^{-1}, \quad (2.3)$$

where C is a numerical constant. Throughout this paper we shall work in units where the Planck length is set equal to one since under a duality transformation the Planck length remains unchanged. If we travel along a closed path around the fundamental string, τ gets transformed by the $\text{SL}(2, \mathbb{Z})$ transformation

$$g_{1,0} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \quad (2.4)$$

We call $g_{1,0}$ the monodromy associated with the fundamental string.

We can combine the $\text{SL}(2, \mathbb{Z})$ symmetry with the existence of the fundamental string to predict the existence of exotic strings [2]. For this consider the effect of $\text{SL}(2, \mathbb{Z})$ transformation on a fundamental string. If there is a fundamental string at coupling $\tau' = (p\tau + q)/(r\tau + s)$, then it should be related to a new string at coupling τ , which we shall call the (s, r) string, with tension

$$T_{s,r} = C \tau_2'^{-1} = C \frac{|r\tau + s|^2}{\tau_2}. \quad (2.5)$$

If we travel along a closed path around the (s, r) string, τ gets transformed by the $\text{SL}(2, \mathbb{Z})$ transformation

$$g_{s,r} \equiv \begin{pmatrix} p & q \\ r & s \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 1+rs & s^2 \\ -r^2 & 1-rs \end{pmatrix}. \quad (2.6)$$

The existence of the monodromy implies that we must identify field configurations related by the duality transformation (2.6) and hence this must be a discrete gauge symmetry. For a generic value of τ this is spontaneously broken since τ is not invariant under this symmetry.

We shall now review how we could measure this monodromy experimentally. The $\text{SL}(2, \mathbb{Z})$ transformation (2.1) acts on the quantized electric and magnetic charge vectors (Q, P) , for appropriate choice of signs of these charges, as

$$\begin{pmatrix} Q \\ P \end{pmatrix} \rightarrow \begin{pmatrix} Q' \\ P' \end{pmatrix} = g \begin{pmatrix} Q \\ P \end{pmatrix}. \quad (2.7)$$

Now consider taking a particle carrying charges (Q, P) and adiabatically transporting it along a closed curve around the string. Since Q, P take values on a fixed lattice, they cannot change continuously and remain fixed during this transport even though τ changes continuously and returns to a value that is related to its original value by the $\text{SL}(2, \mathbb{Z})$ transformation (2.6). To see how the transported particle appears to an observer who has not gone around the string, we need to bring τ back to the original value by the action of the inverse of (2.6). This transforms the charges according to (2.7) with g given by the inverse of (2.6). This can be measured by starting with two particles carrying identical charges, taking one of them around the string and then comparing the charges of the two particles.

In the above discussion we have considered the string to be infinitely long. Such a configuration has infinite mass and changes the asymptotic structure of space-time. So if we want to work in asymptotically flat space-time, we need to consider a loop of this string. If we take the loop to be sufficiently large then locally it will look like a straight string and any experiment we would like to perform on an infinitely long string with finite resources can also be performed on such macroscopic strings, since in any case with finite resources we cannot explore properties of an infinitely long string. In particular the tension (2.3) and the monodromy property (2.6) can be measured using experiments confined near a segment of the macroscopic string. This however relies on one assumption – that the string is accessible to the asymptotic observer and is not hidden behind a black hole horizon. To check this we note that a macroscopic (s, r) string of length L has mass of order

$$L T_{s,r} = C L \frac{|r\tau + s|^2}{\tau_2} \quad (2.8)$$

in Planck units. Hence its Schwarzschild radius $\propto 2m$ is of order

$$R_{s,r} \equiv L \frac{|r\tau + s|^2}{\tau_2}, \quad (2.9)$$

The reason for stating the mass and the Schwarzschild radius as an order of magnitude estimate instead of exact numbers is due to the fact that a loop of string is not spherically symmetric and it acts as a source of the axion-dilaton field besides gravity. Hence we need to take into account the energy density in the axion-dilaton field and the effect of lack of spherical symmetry on the solution. The exact expressions can in principle be calculated by solving the full set of field equations in the presence of a source corresponding to a circular loop of string. Nevertheless it is clear that if $R_{s,r} \ll L$ then the string will be accessible to an asymptotic observer, while if $R_{s,r} \gg L$ then the string will be behind the event horizon and will not be accessible to the asymptotic observer.³ In particular, for the fundamental string $(s, r) = (1, 0)$, and $R_{1,0} = 2CL\tau_2^{-1}$ is much smaller than L in the weak coupling limit $\tau_2 \rightarrow \infty$ and is accessible to the asymptotic observer. However, $R_{1,0}$ is much larger than L in the strong coupling limit and hence the string loop will be behind its event horizon and will not be accessible to the asymptotic observer. More generally, for the (s, r) string, we have $R_{s,r} \ll L$ in the neighborhood of the point $\tau = -s/r$, but for finite τ , we have $R_{s,r} \sim L$. If the asymptotic coupling is weak, i.e. $\tau_2 \gg 1$, then all the (s, r) strings with $r \neq 0$ will have $R_{s,r} \gg L$ and will be behind the horizon.

This raises the question: in what sense the general (s, r) strings exist in an asymptotically weakly coupled theory? To this end, note that it was shown in [11–13] that in an asymptotically flat string theory with a moduli space of vacua, it is possible to create an arbitrarily large space-time region in which the moduli can take any desired value and the curvature, other field strengths and scalar field gradients can be made as small as one likes. In particular, if we take a purely electrically charged near extremal black hole in the heterotic string theory on T^6 , carrying both, momentum and winding charges, along some compact directions, its near horizon geometry has weak string coupling where the fundamental string has low tension. It follows from $SL(2, \mathbb{Z})$ invariance that a dyonic near extremal black hole carrying electric and magnetic charges of the form $(sQ, -rQ)$, will have $\tau \simeq -s/r$ in its near horizon region. By scaling up the mass and charge of such a black hole, one can create an arbitrarily large space-time region where $\tau \simeq -s/r$ so that the (s, r) string is light in this region, and perform

³One could worry whether the energy contained in the fields can make the total energy and hence $R_{s,r}$ proportional to $L \ln L$ [1]. We shall address this in section 5.

experiments on the (s, r) string to study its properties.⁴

In the above discussion we have not discussed in detail the mechanism for producing macroscopic string loops. We take the viewpoint that as long as we are considering a state in the theory it can always be produced, *e.g.* in a high energy collision or the decay of a heavy black hole such states may be produced with small but non-zero probability. For this the (s, r) strings are on the same footing as the fundamental strings since if we can produce macroscopic fundamental string in weakly coupled string theory we can produce (s, r) strings by the S-dual process in the region of space-time where the (s, r) strings are light.

3 $\text{SL}(2, \mathbb{Z})$ multiplet of seven branes in type IIB theory

The case of (s, r) seven branes in type IIB string theory is very similar. The complex coupling is given by

$$\tau = a + \frac{i}{g_s}, \quad (3.1)$$

where a is the asymptotic value of the RR scalar field and g_s is the ten dimensional string coupling. The tension of the D7-brane, also denoted as the $(1, 0)$ brane, is given by

$$\mathcal{T}_{1,0} \propto g_s^{-1} / \ell_s^8. \quad (3.2)$$

In ten dimensions, $\ell_p = g_s^{1/4} \ell_s$. Hence in the Planck units $\ell_s \sim g_s^{-1/4}$ and we get

$$\mathcal{T}_{1,0} = \mathcal{C} g_s^{-1} g_s^2 = \mathcal{C} g_s, \quad (3.3)$$

for some numerical constant $\mathcal{C}$. In terms of τ , this takes the form:

$$\mathcal{T}_{1,0} = \mathcal{C} \tau_2^{-1}. \quad (3.4)$$

The S-duality transformation acts on τ in the same way as (2.1) and produces an (s, r) 7-brane with tension:

$$\mathcal{T}_{s,r} = \mathcal{C} \frac{|r\tau + s|^2}{\tau_2}. \quad (3.5)$$

⁴One could object that to an observer in the near horizon region the (s, r) string will appear as a fundamental string at weak coupling and hence this is not a test of existence of a new kind of string. However asymptotic observers could send charged probe particles to the experimentalists in the near horizon region of the black hole in advance to establish what is electric and what is magnetic charge. After that there will be no ambiguity in distinguishing (s, r) strings from fundamental strings.

A closed loop surrounding a D7-brane has monodromy identical to the one given in (2.4). Using S-duality invariance of the theory one can conclude that an (s, r) 7-brane will produce a monodromy (2.6).

Since flat 7-branes in ten space-time dimensions have infinite mass and changes the asymptotic structure of space-time, they are by themselves not states of the system. However we can construct finite energy states from macroscopic loops of seven branes, where at some given instant in time, the spatial part of the seven brane world-volume lie along a compact subspace of $\mathbb{R}^9$. In particular, we can consider an (s, r) seven brane lying along a seven dimensional sphere of radius L . Up to a numerical factor, its mass will be given by $L^7 \mathcal{T}_{s,r}$ and in 9+1 dimension, its Schwarzschild radius $R_{s,r}$ will be proportional to the 1/7-th power of its mass. This gives

$$R_{s,r} = \mathcal{K} L |r \tau + s|^{2/7} \tau_2^{-1/7}, \quad (3.6)$$

where $\mathcal{K}$ is a numerical constant. From this we see that the spherical (s, r) brane have Schwarzschild radius small compared to the size L of the system only near the region $\tau \simeq -s/r$. In particular in the weak coupling limit $\tau_2 \rightarrow \infty$, all the (s, r) branes other than the D7-brane have Schwarzschild radius much larger than the size L of the system and hence such branes cannot be studied by an asymptotic observer.

The resolution of this issue is also similar to the one used for the four dimensional string theory, namely we can create large regions of space-time where the modulus τ take values close to $-s/r$, and in that region we can have macroscopic loops of (s, r) seven branes and study their properties. The only new ingredient is that the ten dimensional type IIB string theory does not have any charged black holes that can be used to produce such regions. However we can use thick domain walls described in [13] for this purpose.

4 Other codimension two BPS branes

Ref. [10] listed various other BPS exotic branes in string theory. The construction of these branes follows paths similar to the one described in the last two sections, namely in a given compactified string theory, one starts with some known codimension two branes constructed from fundamental strings, D-branes, NS 5-branes or Kaluza-Klein (KK) monopoles and then uses U-duality transformation to construct other codimension two branes. We shall call the original branes the primary branes and the new branes obtained via U-duality transformation the secondary branes.

Now, each of the primary branes have tension small compared to the Planck scale in some region of the moduli space. We have already seen that for D-branes and fundamental strings the tension becomes small when the string coupling becomes small. For NS 5-branes we can make the tension small in Planck units by taking the compact directions transverse to the NS 5-brane to have large size since in this limit the Newton's constant in the compactified theory becomes small and hence the Planck mass becomes large as measured in string scale. For KK monopoles we can make the tension small by taking the circle associated with the KK monopole to have small size. Then by U-duality invariance of the BPS mass formula, measured in the Planck units, it follows that if the primary brane becomes light in some region A of the moduli space, then the secondary brane, obtained from this primary brane by some U-duality transformation g , becomes light in the region $B = g(A)$ of the moduli space, where $g(A)$ denotes g transformation of the region A . We can then create large regions in space-time where the moduli takes value in region B , and in this region a macroscopic loop of the secondary brane will have its Schwarzschild radius small compared to its size. Hence an asymptotic observer will be able to study these branes even if the asymptotic coupling is weak and these branes do not exist in the asymptotic region.

One common feature of the construction of exotic branes described above is that while in a given 'theory' characterized by fixed asymptotic values of the moduli we can produce all BPS exotic branes by producing appropriate values of the moduli fields in different regions of space-time, we cannot necessarily bring these branes together. This avoids some apparent puzzles that would otherwise be present. For example in type IIB string theory, the total monodromy around a (2,1) and (0,1) seven brane is given by:

$$\begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}. \quad (4.1)$$

This is the monodromy around an orientifold seven plane [24]. Thus if we could bring a macroscopic loop of (2,1) and (0,1) seven branes together than they should produce a macroscopic loop of orientifold seven plane. This however leads to a puzzle since the (2,1) and (0,1) seven branes have zero modes corresponding to translation in transverse directions while an orientifold seven plane does not have these modes. Indeed, an orientifold plane has negative tension and a dynamical loop of orientifold plane will render the theory inconsistent. However if (depending on the ambient values of the moduli) either the (2,1) seven brane or the (0,1) seven brane is hidden behind an event horizon, then the whole system will be behind an event horizon and there is no contradiction.

5 Scaling of macroscopic codimension two brane loops

In the previous sections we have discussed how by creating large regions in space-time where the brane tension can be made small we can ensure that a loop of the brane is not hidden by an event horizon. In order to study the properties of an isolated flat brane we also need to be able to make the size of the loop arbitrarily large so that locally a portion of the brane is indistinguishable from that of an infinite flat brane. This can be done using the same scaling transformation that was used in [11–13] to create an arbitrarily large region in space-time where the moduli take values different from their asymptotic values. However, since for codimension two branes the metric and various scalar fields vary logarithmically as a function of the radial distance from the brane, one might worry if there is any violation of the scaling property due to this logarithmic dependence [1]. In this section we shall discuss why this is not the case, at least for the BPS branes that we have discussed above.

For definiteness we shall illustrate this using the solution describing a fundamental string in four dimensional heterotic or type II string theory. The solution describing the infinitely long string was given in [25] and may be expressed as,

$$ds_c^2 = -dt^2 + dz^2 + g_4^{-2} \left(1 - \frac{g_4^2}{2\pi} \ln \frac{r}{L_0} \right) (dr^2 + r^2 d\theta^2), \quad \tau = i g_4^{-2} \left(1 - \frac{g_4^2}{2\pi} \ln \frac{r}{L_0} e^{i\theta} \right), \quad (5.1)$$

where ds_c^2 is the four dimensional Einstein frame metric and τ is the axion-dilaton field whose real part measures the scalar obtained by dualizing the two form gauge field and whose imaginary part measures the inverse of the square of the string coupling. g_4 and L_0 are parameters of the solution. We shall take g_4 to be small but finite. We shall see that the parameter L_0 is associated with the freedom of scaling transformation described in (5.2). As expected, τ is shifted by 1 as we go around the string and θ changes by 2π . Near the core $r \rightarrow 0$, $\tau \rightarrow i\infty$ and hence the string coupling is weak, but the solutions hits a singularity when $\frac{g_4^2}{2\pi} \ln \frac{r}{L_0} = 1$. For the string loop solution of the type we shall consider, we shall never hit this singularity.

Now instead of considering an infinitely long string, let us consider a loop of coordinate radius L_0 . For definiteness, let us take the string to be in the $\theta = 0$ plane, passing through the point $r = 0$, as shown in Fig. 1. Then for $r \ll L_0$, $|z| \ll L_0$, the solution will look approximately as in (5.1), but the logarithmic growth for large r gets cut off at $r \sim L_0$. Beyond this distance the scalar fields will evolve according to the three dimensional Laplace equation and approach a constant up to corrections that fall off as inverse power of the distance from the loop. As a result the metric at the center of the loop and at infinity will be approximately

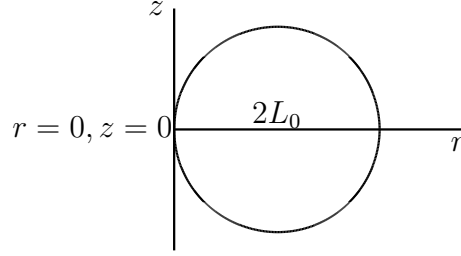


Figure 1: A string loop of coordinate radius L_0 in the $\theta = 0$ plane.

$\eta_{\mu\nu}$ after a rescaling of r by g_4 , and the imaginary part τ_2 of τ at the center of the loop and at infinity will be approximately g_4^{-2} . The solution will also evolve in time over a time scale of order L_0 .

We now apply the scaling transformation on this solution as follows. In any general coordinate invariant theory in D space-time dimensions, where each term in the action has two derivatives, there is a standard scaling property under which a covariant tensor field $C^{(k)}$ of rank k , a contravariant tensor field $B^{(k)}$ of rank k and the action S scales as follows [13, 26]:

$$C^{(k)} \rightarrow \lambda^k C^{(k)}, \quad B^{(k)} \rightarrow \lambda^{-k} B^{(k)}, \quad S \rightarrow \lambda^{D-2} S. \quad (5.2)$$

In particular the metric $g_{\mu\nu}$ scales as λ^2 and the scalars remain invariant. This converts (5.1) to

$$ds_c^2 = -\lambda^2 dt^2 + \lambda^2 dz^2 + \lambda^2 g_4^{-2} \left(1 - \frac{g_4^2}{2\pi} \ln \frac{r}{L_0} \right) (dr^2 + r^2 d\theta^2), \quad \tau = i g_4^{-2} \left(1 - \frac{g_4^2}{2\pi} \ln \frac{r}{L_0} e^{i\theta} \right), \quad (5.3)$$

Since the metric is scaled by an overall constant, the new solution does not have any event horizon since the original solution did not have one. Under this scaling, the interval over which the solution appears to be a straight string scales as λ and this can be taken to be as large as we like by taking λ large. Also the physical time over which the solution evolves will scale as λ , showing that for large λ the evolution slows down. We can introduce new coordinates $(t', z', r') = \lambda(t, z, r)$ so that the asymptotic metric is given by $\eta_{\mu\nu}$ after a further rescaling of r' by g_4 . This effectively replaces L_0 by λL_0 . In the new coordinate the string will appear to be straight in the region $|z'| \ll \lambda L_0$ and $|r'| \ll \lambda L_0$. Similarly the time interval $\Delta t'$ over which the solution evolves will also be large, carrying a factor of λ . Finally τ_2 is still of order g_4^{-2} at infinity and at the center of the loop and τ_1 changes by 1 as we go once around the

string, showing that the new solution still represents a single fundamental string.⁵ This allows us to study the properties of a straight string of any desired length provided we have enough resources to produce a large string loop of this type.

One can make this discussion more explicit as follows. Let us consider a circular loop of radius L at rest at $x^0 = 0$ and let it evolve in time. If we take the string to lie in the x^1 - x^2 plane centered at $x^1 = 0$, $x^2 = 0$, then the subsequent evolution of the classical string is given in the parametric form as [27]:

$$X^0 = L\xi, \quad X^1 = L \cos \xi \cos \sigma, \quad X^2 = L \cos \xi \sin \sigma. \quad (5.4)$$

In this coordinate system, the scaling transformation (5.2) can also be described as the scaling of L by λ keeping the metric fixed. Now suppose that we want to study the properties of the string by staying at a distance ℓ_0 from the string for a period t_0 . Our goal will be to show that for any given ℓ_0 and t_0 , we can ensure that the displacement of the string remains small compared to ℓ_0 during the time t_0 so that the string appears static during the experiment. For this we begin the experiment at $x^0 = 0$ at the point $x^2 = 0$, $x^1 = L - \ell_0$ so that the point on the string that is nearest to the experimentalist corresponds to $x^1 = L$, $x^2 = 0$. At $x^0 = t_0$ this will reach the point

$$\xi = t_0/L \quad \Rightarrow \quad x^1 = L \cos(t_0/L) \simeq L - t_0^2/(2L). \quad (5.5)$$

This shows that however large a time t_0 we need for the experiment, by taking L to be sufficiently large we can ensure that the string remains almost static during the experiment. In particular, to measure the monodromy around the string remaining at a distance ℓ_0 we need a time of order $2\pi\ell_0$, during which the string can be made to remain almost static, provided ℓ_0 does not scale with L .

Even though the argument based on the scaling transformation is correct, it is instructive to examine in some detail possible sources of logarithmic violation since this has potential consequences for conjectured non-BPS branes for which the analog of the solution (5.1) is not known. For this we write the expression for τ given in the scaled coordinates described below (5.3) as

$$\tau = ig_4^{-2} - \frac{i}{2\pi} \ln \frac{w}{L}, \quad w \equiv r' e^{i\theta}, \quad L \equiv \lambda L_0, \quad (5.6)$$

and try to compute the energy per unit length of the string due to the scalar kinetic term by taking an upper cut-off of L on $|w|$. If τ had been a canonically normalized scalar, then the

⁵By contrast, the number of branes of other codimensions scale by some power of λ under the scaling [13].

energy of the field per unit length will be proportional to

$$\int d^2w \partial_w \tau \partial_{\bar{w}} \bar{\tau} = \frac{1}{2\pi} \int dr' r'^{-1}. \quad (5.7)$$

This requires both an upper and a lower cut-off to get a finite answer. The upper cut-off is the size L of the loop since beyond this distance the scalar field will evolve according to the three dimensional Laplace equation and approach a constant up to corrections that fall off as inverse power of the distance from the loop. The lower cut-off may be taken to be some microscopic scale μ . This gives the energy per unit length contained in the scalar field to be $\ln(L/\mu)$ and the total energy to be $L \ln(L/\mu)$, contradicting the claim that the energy scales as L [1]. The scaling argument fails since the microscopic cut-off implicitly requires terms in the action other than those containing only two derivatives.

In actual practice, the kinetic term of τ has an additional factor of τ_2^{-2} . With this the energy per unit length contained in the scalar field becomes proportional to

$$\int d^2w \tau_2^{-2} \partial_w \tau \partial_{\bar{w}} \bar{\tau} = \frac{1}{2\pi} \int dr' r'^{-1} \left(g_4^{-2} - \frac{1}{2\pi} \ln \frac{r'}{L} \right)^{-2} = \left(g_4^{-2} - \frac{1}{2\pi} \ln \frac{r'}{L} \right)^{-1} \Big|_{\mu}^L. \quad (5.8)$$

Up to a numerical factor, g_4 can be equated to the asymptotic value of the string coupling. Then (5.8) may be written as

$$g_4^2 - \left(g_4^{-2} + \frac{1}{2\pi} \ln \frac{L}{\mu} \right)^{-1}. \quad (5.9)$$

We now see that the second term vanishes in the limit $\mu \rightarrow 0$ leaving us with the first term g_4^2 . Therefore the result is independent of L , in agreement with the scaling laws. The important observation here is that the result is not sensitive to the microscopic scale μ . Had it been otherwise, we could not have applied the scaling argument since the microscopic cut-off scale will violate scaling laws.

The analysis described here can be generalized to other BPS exotic branes of the type discussed in section 2, 3 and 4. In particular the scalar field produced by these branes take the same form as (5.1), with τ having different interpretation for different branes. For example for the D7-brane τ is the axion-dilaton modulus of the ten dimensional type IIB string theory, while for the NS 5-brane with two transverse directions compactified on a T^2 , τ is the complexified Kahler modulus associated with the T^2 . But in all such cases the analogs of (5.1) and (5.8) hold, with g_4 having different interpretations, and ensures that the dependence on the microscopic

cut-off disappears, restoring the scaling law. This is easiest to see in type II string theory on T^6 where the fundamental string is related to various exotic strings by various duality transformations and we can construct the solution describing these exotic strings by duality transformation of the solution (5.1). Once these solutions have been constructed, a subset of them carrying NSNS charges can also be regarded as solutions in heterotic string theory on T^6 .

To contrast this with general codimension two branes, let us compute the energy of a vortex string solution in a field theory of a complex scalar field with potential $\lambda(\phi^*\phi - a^2)^2$. In this case we can construct a solution with scalar field profile $\phi = f(r)e^{i\theta}$ with $f(r) \rightarrow a$ for $r \gg \mu \equiv 1/(a\sqrt{\lambda})$ and $f(0) = 0$. The scalar kinetic term then produces an energy per unit length of the string of order

$$\int 2\pi r dr (\partial_r \phi^* \partial_r \phi + r^{-2} \partial_\theta \phi^* \partial_\theta \phi) \simeq 2\pi \int_\mu^L dr r^{-1} a^2 \simeq 2\pi |a|^2 \ln(L/\mu), \quad (5.10)$$

where we have put an upper cut-off L on the r integral by taking the string to be a macroscopic loop of length L . We see that this violates the scaling law. This can be traced to the dependence of the result on the microscopic cut-off scale μ . We have discussed this example here, since while studying the energy of macroscopic loops of non-BPS branes, we need to ensure that such logarithmic dependence on the size L of the loop is absent.

6 U-duality as gauge symmetries

It is generally believed that once we take into account the effects of quantum mechanics and gravity, there are no global symmetries. In this section we shall discuss the role of codimension two branes in establishing that the U-duality symmetries of string theory are (spontaneously broken) discrete gauge symmetries. As in the rest of the paper, our analysis will apply only to asymptotically flat space-times.

Before discussing whether the duality transformations are global or gauge symmetries, let us first discuss whether they are symmetries at all. At a generic point in the moduli space the U-duality symmetries are spontaneously broken by the asymptotic values of the moduli. Nevertheless, as discussed in [11–13], in the class of theories we are analyzing, given any set of asymptotic values of the moduli collectively labelled as A, we can create an arbitrarily large region $\mathcal{R}$ of space-time in which the moduli take any other set of values B. Therefore, if we choose B and A to be related by a U-duality transformation then U-duality symmetry predicts

the result of any experiment in the region $\mathcal{R}$ by knowing the corresponding results in the asymptotic region, This is clearly a testable prediction and identifies U-duality transformations as spontaneously broken symmetries.

In order to distinguish between gauge and global symmetries, we shall take a practical approach as seen from an experimentalist's viewpoint instead of the more formal and rigorous approach adopted in [15, 16]. Given a transformation g , we shall say that it is a gauge symmetry if two field configurations related by g are identified. Hence we should allow classical configurations (coherent states) with the property that in that state, as we go around a closed loop, the fields undergo a g transformation [1, 28].⁶ Conversely, the existence of such a state in the theory will imply that the transformation under consideration is a gauge symmetry. In this context we also note that whether a given transformation is a gauge symmetry or not could depend on the string compactification we have, – establishing that a transformation is a gauge symmetry in one compactification does not establish that it is a gauge symmetry in another compactification unless in the latter theory we can create an arbitrarily large region in space-time inside which we have the former theory.

We shall now try to make this requirement more precise. A transformation g can be called a gauge symmetry if the system has a (time dependent) state that has two time-like curves C_1 and C_2 , beginning and ending at the same points A and B in space-time, with the following properties:

1. Both curves lie in regions of space-time where, within the desired precision, the environment is locally indistinguishable from the vacuum associated with the values of the moduli scalar fields in that region. In particular the components of the stress tensor in the rest frame of the particle moving along C_1 or C_2 , multiplied by the proper time along the trajectory, must be small so that the integrated effect of departure from the vacuum along the trajectories remain small. This does not exclude (scalar) fields that vary along the trajectory and undergo monodromy, but the stress tensor produced by the varying fields must satisfy the requirement described above.
2. Since the trajectories may have to undergo acceleration, we need to ensure that the

⁶This is sufficient to establish that a given transformation is a gauge symmetry, but there may be other ways to do this. Nevertheless it is a very useful tool, since in a theory of gravity measurements are always more subtle due to the possibility of black hole formation. For example a measurement that requires us to simultaneously place finite mass detectors covering the whole solid angle around a point may not be possible since such a system of detectors will have a large enough mass so that they form a blackhole whose horizon surrounds them.

acceleration measured in the rest frame of the particle is small so that a test particle moving along such a trajectory effectively behaves as if it is moving in vacuum. This will also ensure that the effect of Unruh radiation experienced by a particle moving along the trajectory remains small.

3. If we take a localized quantum state at A , e.g. a particle, and transport it to B , then the state at B transported along C_1 differs from the state at B transported along C_2 by the transformation g .

Conditions 1 and 2 on the paths are important since they ensure that the transformation by g is not due to any other dynamical effect like scattering with another particle.

First we review the case of a quantum field theory without gravity. Let us take the rigid part of the $U(1)$ gauge symmetry in 3+1 dimensions under which a charged particle picks up a phase. In this case we can consider a solenoid carrying a magnetic field and C_1 and C_2 can be taken to be time-like curves from A to B on two sides of the magnetic flux lines. If we transport a quantum state at A to B , then the state transported along C_2 and a state transported along C_1 will have a relative phase due to Aharonov-Bohm effect even though neither path encounters any magnetic field and locally experiences vacuum. Of course when full quantum effects are taken into account the effect of the flux will be felt outside the solenoid and neither path will be really in the vacuum, but by taking the size of the system to be very large keeping the cross section of the solenoid fixed and the paths to be far away from the solenoid, we can reduce this effect to any desired degree.

Once the effect of gravity is taken into account there are further complications. First of all, due to the long range nature of gravity and the constraints, even classically we cannot switch off the gravitational field outside a given region carrying non-vanishing energy density. Second, since the metric fluctuates, there is no gauge invariant notion of a fixed curve in space-time. For this reason the conditions 1 and 2 above become more important, since only if the local environment of every point on the trajectory is approximately that of the vacuum, we can ignore the classical background gravitational field and the effects of quantum gravitational fluctuations for low energy processes. This can be achieved by using the scaling transformation by λ described in (5.2) to generate a new solution from a given solution and maps classical particle trajectories to new trajectories. Under this scaling the proper time along the trajectory scales as λ while all the scalars constructed from $2n$ derivatives of fields scale as λ^{-2n} and becomes small for large λ for $n \geq 1$. With a little work one can also show that the energy

momentum tensor $T_{\mu\nu}$ (including the gravitational contribution), measured in the rest frame of the particle in which the local metric is $\eta_{\mu\nu}$, the first derivative of the metric is zero and the particle is at rest, scales as λ^{-2} in the large λ limit. Therefore not only locally the background appears to be that of the vacuum, but the integrated effect of the departure from the vacuum over the entire trajectory falls at least as fast as λ^{-1} for large λ and hence vanishes in the $\lambda \rightarrow \infty$ limit. One can also show that the acceleration of the particle at any point on the trajectory, measured in the rest frame of the particle, scales as λ^{-1} . Hence the Unruh temperature scales as λ^{-1} and the Unruh radiation density scales as λ^{-D} . So even the integrated effect of Unruh radiation encountered by the particle vanishes as λ^{1-D} , and the particle moving along the trajectory can be made to feel that it is in the vacuum to any desired degree of accuracy. Since the particle always sees the local environment as the vacuum, we expect that the semiclassical notions like particle trajectories still make sense.

One can now see how the existence of (s, r) strings in heterotic string theory in $T^6 \times \mathbb{R}^{3,1}$ and (s, r) seven branes in type IIB string theory in $\mathbb{R}^{9,1}$ can be used to show that the corresponding $\text{SL}(2, \mathbb{Z})$ symmetries in these theories are gauge symmetries. By taking $C_1 - C_2$ to be a curve that links the world-volume of the string or the seven brane, one sees that the state transported along C_1 and along C_2 differ by the $\text{SL}(2, \mathbb{Z})$ transformation (2.6). Furthermore by using the scaling transformation (5.2) and taking λ to be large one can ensure that the deviation from vacuum along the trajectory remains small. This establishes that $\text{SL}(2, \mathbb{Z})$ transformations of the form (2.6) are gauge symmetries. This is sufficient for establishing that all $\text{SL}(2, \mathbb{Z})$ transformations are gauge symmetries, since the matrices $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, – the monodromy matrices associated with the $(1,0)$ and $(0,1)$ strings, – generate the whole S-duality group. This does not mean that we can construct a single string or seven brane that produces the desired $\text{SL}(2, \mathbb{Z})$ monodromy. However by threading a path through different (s, r) string / seven brane loops, situated in different regions in space-time where their tensions are small, we can produce the desired monodromy.

A similar analysis can be done for the T-duality symmetries in heterotic string theory on $T^6 \times \mathbb{R}^{3,1}$. For example an NS 5-brane wrapped on four of the six directions of T^6 will produce a monodromy that shifts the component of the 2-form field along the other two directions of the torus by one unit. Various other (exotic) codimension two branes related to this by T-duality transformation produces other monodromies in the T-duality group, and together they establish that the whole T-duality group is a spontaneously broken discrete gauge symmetry. More generally in any string compactification, monodromy around the exotic branes can be

used to establish that the discrete U-duality groups of those theories are also (spontaneously broken) gauge symmetries.

Note that in the heterotic string theory on $T^6 \times \mathbb{R}^{3,1}$, as we transport a probe charged particle around an (s, r) string, the charge carried by the probe changes by the monodromy transformation. Since the total charge is conserved, the extra charge must be deposited on the string. A simple example of this is the transport of a KK monopole around a fundamental string. By the rules of duality transformation, the KK monopole picks up one unit of fundamental string winding charge along the compact direction. This is achieved by the macroscopic fundamental string undergoing a wrapping along the compact direction during the transport of the KK monopole around it. A general (s, r) string will pick both electric and magnetic charges under such processes. Their ability to absorb the charge is encoded in the existence of appropriate zero modes on these strings, whose excitations allow the string to carry the charges that they are expected to absorb during the transport of dual charged particles around them [2].

7 AdS vs flat space-time

The results of the previous section may appear to contradict what we know from anti-de Sitter space-time where the spontaneously broken gauge symmetries in the bulk theory of gravity have no natural manifestation in the dual boundary theory [15, 16]. This can be illustrated in the celebrated example of the duality between N=4 supersymmetric Yang-Mills theory and type IIB string theory on $AdS_5 \times S^5$ [29]. The type IIB string theory has $SL(2, \mathbb{Z})$ S-duality symmetry that is spontaneously broken at a generic point in the moduli space. In the dual boundary theory this is reflected in the fact that the $\mathcal{N} = 4$ supersymmetric Yang-Mills theories at two different values of the complex coupling τ are equivalent if the corresponding values of τ are related by $SL(2, \mathbb{Z})$ transformation [30, 31]. Hence for a generic value of τ the duality symmetry is explicitly broken in the boundary theory, and does not lead to any direct experimental consequences. So if an experimentalist in the bulk can verify the existence of this symmetry by measuring monodromy around an (s, r) 7-brane, this would lead to a contradiction.

The resolution of this puzzle seems to lie in the difference in our ability to measure the monodromy around a 7-brane in flat space-time and in AdS space-time. In flat space-time, a 7-brane loop of size L can last for a time of order L that can be made as large as we want. In

AdS space-time the scaling ‘symmetry’ (5.2) is broken due to the presence of the cosmological constant, and cannot be used to generate new solutions from old ones. More specifically, in AdS space-time such a seven brane loop will collapse in a time scale set by the AdS scale [13]. Hence we cannot do a controlled experiment on the seven branes for arbitrarily large time and determine the monodromy around them to the desired level of confidence. In fact, in AdS space-time we do not even have a controlled test of the duality transformations being symmetries at all, since we cannot create arbitrarily large spatial regions, lasting for arbitrarily long time, inside which the moduli take values that are different from their asymptotic values [13]. This leads to a consistent conclusion that the duality symmetry does not have any observable consequences either in the boundary theory or in the bulk theory. Nevertheless, for $SU(N)$ super-Yang-Mills theory with coupling g_{YM} , this property must emerge in the limit of large N at fixed g_{YM} , since in this limit $AdS_5 \times S^5$ approaches flat space-time.

8 Non-BPS branes

So far we have discussed BPS exotic branes. String theory also has conjectured non-BPS exotic branes for which we do not know the tension. However if one accepts the criterion for the existence of a codimension two brane described above, one can propose the following as the requirement for the existence of a non-BPS exotic brane:

A necessary criterion for a non-BPS exotic brane to exist in a theory is that its tension is small enough in some corner of the moduli space so that a large spherical brane is not hidden behind its Schwarzschild radius.

We shall now discuss some examples of conjectured codimension two non-BPS branes. The first example is the reflection seven brane (R7-brane) discussed in [32–35]. These are conjectured codimension two branes in type IIA and type IIB string theories with the property that as we transport a state around such a brane, the state returns transformed by a $(-1)^{F_L}$ transformation. Here $(-1)^{F_L}$ is the $\mathbb{Z}_2$ transformation that changes the signs of all the RNS and RR states of the theory. The existence of these branes would establish in particular that $(-1)^{F_L}$ is a gauge symmetry. From our earlier analysis of codimension two branes it follows that in order for an R7-brane to exist in the strong sense described above, there must be regions in the moduli space where the brane tension is sufficiently small so that a macroscopic loop of the R7-brane will not be behind its own event horizon.⁷ While this constraint is not as severe

⁷This is not just a condition on the tension but also involves information on what other sources of massless

as the one for codimension one brane, this still puts a strong restriction on the tension of the brane.

Since the R7-brane has a monodromy of $(-1)^{F_L}$, all RR fields must change sign as we go once around the loop. This applies to the RR scalar field as well. So if we place an R7-brane loop in a background where the RR scalar field takes a non-zero value a_0 , then it must vary as we go around the R7-brane, having the form $a_0 e^{i\theta/2}$ far away from the brane. If the dilaton remains constant, then the kinetic term of the RR scalar will give an energy of order (5.7), leading to scaling violation. So either the dilaton must vary, tempering the divergence as in (5.8), or we have to conclude that a macroscopic loop of R7-brane can only exist in regions of space-time where the RR scalar vanishes.

As a side remark, we note that the existence of R7-brane by itself is not needed for establishing that $(-1)^{F_L}$ is a gauge symmetry. What one needs is a combination of 7-branes, possibly existing in different regions of space-time with different values of moduli, such that we can find a path looping around these branes along which the total monodromy is $(-1)^{F_L}$. For example the monodromy around a D7-brane, acting on the NSNS 2-form field and RR 2-form field, is given by $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. So if we find another brane that produces a monodromy $\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$ then the product of the two will give

$$\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (8.1)$$

which is precisely $(-1)^{F_L}$ transformation. Therefore the existence of a brane with monodromy $\begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix}$ will also be sufficient to establish that $(-1)^{F_L}$ is a gauge symmetry.

Another example of a $\mathbb{Z}_2$ symmetry in string theory, that is not obviously a gauge symmetry, is the exchange of two E_8 factors in the $E_8 \times E_8$ heterotic string theory in ten space-time dimensions. If we had a (non-BPS) 7-brane with the property that as we go around the 7-brane the two E_8 factors get exchanged and the tension of the brane is sufficiently small in some corner of the moduli space so that a loop of the brane is not hidden behind its event horizon, then it would establish that this symmetry is a gauge symmetry. The world-sheet description of such branes was proposed in [36, 37], but it is not known if large loops of these branes can avoid being hidden behind an event horizon. Similar questions can be asked for other known symmetries in string theory, *e.g.* the ones analyzed in [38].

fields the brane provides, since whether the macroscopic loop will be behind the horizon is determined by a combination of all these data.

There is also a conjectured codimension one brane in ten dimensional string theory that is supposed to separate the vacua of type IIA and type IIB string theory [39]. If we use the strong criteria of their existence, namely that we should be able to create arbitrarily large size loops of these branes so that it looks locally flat and study their properties, then it follows from the discussion in section 1 that this brane exists only if its tension, measured in Planck units, vanishes in some corner of the moduli space. Since this cannot happen in the weak coupling limit of either string theory where we know the spectrum of the theory, this should happen at some finite value g_0 of the coupling. In that case we can produce a large region in space-time where the coupling is arbitrarily close to g_0 and study these branes. Vanishing of the tension at some point in the moduli space is clearly a strong constraint on the brane.

8.1 Infinite tension branes

Refs. [36,37] proposed the existence of some infinite tension branes in ten dimensional heterotic string theories. These have codimension $(n + 1)$ with $n = 4$ and 8 , i.e. they are 4-branes and 0-branes. They are sources of 1-form gauge field, with the property that at a distance r away from the brane, the field strength falls off as $1/r^2$. Therefore the energy density falls off as $1/r^4$ and the energy per unit volume of the brane has a term proportional to

$$\int^{\infty} r^n dr \times r^{-4}. \quad (8.2)$$

This diverges for $n = 4$ and $n = 8$.

Now, even for finite tension, infinite branes have infinite energy (except for 0-branes) and hence are not regular states in the theory. Having infinite tension makes things worse. However the relevant question, both for finite and infinite tension branes, is whether we can construct a spherical brane of radius L that has energy that grows slower than L^7 for large L .⁸ In that case its Schwarzschild radius would grow slower than L and the brane would not be shielded by an event horizon. We have already seen that for finite tension branes this condition is satisfied for branes of codimension larger than two. For a spherical infinite tension brane of the type described in [36,37], the gauge field strength will fall off as r^{-2} up to a distance L and fall off much faster beyond this distance since the gauge field configuration on the celestial sphere becomes topologically trivial. Such brane configuration will have total energy density

⁸For 0-branes this constitutes a brane anti-brane pair separated by a distance $2L$.

proportional to

$$\int^L r^8 dr \times r^{-4} \sim L^5. \quad (8.3)$$

This shows that indeed the total energy grows slower than L^7 and the would be Schwarzschild radius grows as $L^{5/7}$. Since for large L this is less than L , we see that the brane is not shielded from the asymptotic observer by an event horizon.

By taking L sufficiently large we can ensure that locally the brane looks like a flat brane to any desired accuracy. Whatever property of a flat brane of this type we want to study can now be studied by performing experiments on such branes.

9 Exposing the exotic branes from compact dimensions

So far we have discussed the possibility of studying macroscopic loops of exotic branes. However, exotic branes are also present in many string compactifications [18–20, 40, 41]. These fill the non-compact part of the space-time but may be localized along some of the compact directions. At a generic point in the moduli space of the theory the sizes of the transverse directions are small and we cannot isolate these branes and study them. However in special regions of the moduli space where the transverse directions become large, the exotic branes may get isolated from the rest of the system and one can perform experiments to study their properties.

We note, however, that the existence of these branes cannot be used to argue that the monodromy around such a brane is a gauge symmetry in the higher dimensional theory that gets exposed. For this we have to be able to create loops of these branes in the higher dimensional theory itself. From this perspective, the goal of this section is somewhat different from that in the earlier sections. Instead of using exotic branes to show that certain transformations are gauge symmetries, we simply explore exotic branes in asymptotically weakly coupled string compactifications.

9.1 (s,r) seven branes from F-theory compactification

In this subsection we shall illustrate the procedure in the context of one particular example: F-theory on $K3 \times T^4$ [18]. We can describe this compactification as type IIB string theory compactified on the product of a two dimensional sphere B times a four dimensional torus T^4 of finite size (in string scale), with twenty four (s, r) seven branes localized at various points

on the sphere B and extending along $T^4 \times \mathbb{R}^{3,1}$. When the size of the sphere B becomes large then the 7-branes get exposed and can be studied by an asymptotic observer. Our goal will be to examine how this can be achieved in a theory where the size of B is small asymptotically.

The theory described above is in the same moduli space as heterotic string theory on T^6 . We shall first identify the region in the moduli space in which the F-theory description is valid, For this we note that this theory also has a description as a orientifold of type IIB on $T^2 \times T^4$ [24], where the orientifold group acts by reversing the sign of the two directions along T^2 together with the internal symmetry $(-1)^{F_L}\Omega$ where $(-1)^{F_L}$ is the symmetry that changes the sign of all the RNS and RR states and Ω is the world-sheet parity transformation. The sphere B is identified as the space $T^2/\mathbb{Z}_2$, where $\mathbb{Z}_2$ acts by reversing the sign of the two directions of T^2 . This produces four orientifold seven planes localized at the four fixed points on T^2 . To cancel the RR charges carried by the orientifold seven planes we need to add 16 D7-branes placed at various points on $T^2/\mathbb{Z}_2$ and extending along $T^4 \times \mathbb{R}^{3,1}$. Once non-perturbative effects are taken into account, each orientifold plane splits into a pair of (s, r) seven branes for appropriate (s, r) . This gives altogether 24 seven branes which can be identified as those required for an F-theory compactification. If we place 4 D7-branes on each of the four orientifold planes, then the split in the orientifold also disappears and we get $SO(8)^4$ unbroken gauge group, one from each orientifold plane. We shall assume that the configuration is such that the $SO(8)^4$ is slightly broken to $U(1)^{16}$, so that the D7-branes are not exactly on top of the orientifold plane but they are close enough so that the analysis of [24] holds. In the limit where the size of $B = T^2/\mathbb{Z}_2$ becomes large keeping the Wilson lines fixed (so that the ratios of gauge boson masses remain fixed) the distances between the seven branes increase and we can study them individually.

We shall now examine what this limit correspond to in the heterotic variables. In the type IIB description, let $\tilde{R}_B$ denote the order of the radii of the four circles of T^4 which we shall label by coordinates x^4, x^5, x^6, x^7 and let R_B denote the order of the radii of the two circles of T^2 which we label by x^8, x^9 , all measured in the type IIB string metric:

$$R_{4B} \sim R_{5B} \sim R_{6B} \sim R_{7B} \sim \tilde{R}_B, \quad R_{8B} \sim R_{9B} \sim R_B. \quad (9.1)$$

Also let g_B be the coupling constant of ten dimensional type IIB theory.⁹ We shall follow the

⁹Note that in the F-theory limit the type IIB coupling varies over the sphere B except in the special case when each orientifold plane has 4 D7-branes on it. When we consider the case where each orientifold plane has four D7-branes near it, g_B can be taken to be the value of the coupling away from the orientifold D7-brane system.

convention that a string coupling without superscript will always denote the ten dimensional string coupling while the four dimensional coupling will carry a superscript (4). We now make T-duality transformation along the 8 and 9 directions to convert this into type I string theory on $T^2 \times T^4$, with the gauge group $SO(32)$ broken to $U(1)^{16}$ by Wilson lines along the 8 and 9 directions. The ten dimensional coupling constant g_I of the type I string theory and the radii R_I of the 8,9 directions and $\tilde{R}_I$ of the 4,5,6,7 directions measured in the type I metric are given by

$$\tilde{R}_I \sim \tilde{R}_B, \quad R_I \sim 1/R_B, \quad g_I \sim g_B R_B^{-2}. \quad (9.2)$$

In ten dimensional Planck units, the radii $\tilde{r}$ of the 4,5,6,7 directions and r of 8,9 directions are given by

$$\tilde{r} \sim \tilde{R}_I g_I^{-1/4} \sim \tilde{R}_B R_B^{1/2} g_B^{-1/4}, \quad r \sim R_I g_I^{-1/4} \sim R_B^{-1/2} g_B^{-1/4}. \quad (9.3)$$

In the heterotic description the radii $\tilde{R}_H$ of the 4,5,6,7 directions, R_H of 8,9 directions and the ten dimensional coupling g_H are given by

$$g_H \sim 1/g_I \sim g_B^{-1} R_B^2, \quad \tilde{R}_H \sim \tilde{r} g_H^{1/4} \sim \tilde{R}_B R_B g_B^{-1/2}, \quad R_H \sim r g_H^{1/4} \sim g_B^{-1/2}. \quad (9.4)$$

From this we get the four dimensional heterotic string coupling to be

$$g_H^{(4)} \sim g_H \tilde{R}_H^{-2} R_H^{-1} \sim g_B^{1/2} \tilde{R}_B^{-2}. \quad (9.5)$$

Finally, inverting (9.4), (9.5) we get

$$g_B \sim R_H^{-2}, \quad \tilde{R}_B \sim \left(g_H^{(4)}\right)^{-1/2} R_H^{-1/2}, \quad R_B \sim \tilde{R}_H R_H^{-1/2} \left(g_H^{(4)}\right)^{1/2}, \quad (9.6)$$

As in the original description, the $SO(32)$ gauge group is broken to its $U(1)^{16}$ subgroup by Wilson lines along the 8 and 9 directions.

Since to expose the 7-branes in the F-theory description we need R_B and $\tilde{R}_B$ to be large, with g_B finite, we see from (9.4), (9.5) that this translates to finite R_H , small $g_H^{(4)}$ and large $\tilde{R}_H$. We shall now demonstrate how we can create a large region near the horizon of a suitable black hole where the moduli take such values, in a theory where the asymptotic moduli take finite values.¹⁰ For simplicity we shall assume that the asymptotic moduli are such that the appropriate Wilson lines along the 8 and 9 directions are already turned on and 4,5,6,7

¹⁰It was shown in [13] that by considering a system of nested black holes we can access any point in the moduli space of heterotic string theory on T^6 from any given point. Here the goal is to show that the F-theory configuration can be achieved by a single black hole. In some sense, higher dimensional theories have less moduli and hence are easier to reach.

directions are described by the product of four circles at their self-dual radii, and are orthogonal to the 8 and 9 directions. These assumptions are not necessary but will simplify the solution. We now consider a purely electrically charged black hole in this theory carrying momentum charges along the 4,5,6,7 directions. This corresponds to setting in the solution (A.1) of [12], $\beta = -\alpha$ and $\vec{p} = \vec{n}$ for the first four components, with the rest of the components set to zero. The quantity of interest is the heterotic string metric G on T^6 , and the four dimensional string coupling $g_H^{(4)}$, which are given by

$$G = I_6 + \frac{2m}{\rho} \sinh^2 \alpha \, n n^T, \quad g_H^{(4)} = g_0^{(4)} \left(1 + \frac{2m}{\rho} \sinh^2 \alpha \right)^{-1/4}, \quad (9.7)$$

near the horizon of the black hole. Here ρ is the radial coordinate, $g_0^{(4)}$ is the asymptotic value of the heterotic string coupling, and m , α and the six dimensional unit vector $\vec{n}$ are parameters labelling the mass and charges of the solution. The horizon is situated at $\rho = 2m$, and we are considering a region where $\rho \sim m$ but $\rho > 2m$ so that the region is outside the horizon. Following [12] we take α large, corresponding to a near extremal black hole, and

$$\vec{n} = (1, N^{-1}, N^{-2}, N^{-3}, 0, 0) / \sqrt{1 + N^{-2} + N^{-4} + N^{-6}}, \quad (9.8)$$

for some large integer N . Consider now a cycle in T^6 that joins the point $(0, 0, 0, 0, 0, 0)$ to $2\pi(k_1, k_2, k_3, k_4, k_5, k_6)$ in the $(x^4, x^5, x^6, x^7, x^8, x^9)$ directions for some integers $k_1, k_2, k_3, k_4, k_5, k_6$. The length of this cycle is given by,

$$L(\vec{k}) = 2\pi \sqrt{\vec{k}^2 + \frac{2m}{\rho} \sinh^2 \alpha (\vec{k} \cdot \vec{n})^2}. \quad (9.9)$$

Let us choose ρ , α and N such that

$$\frac{2m}{\rho} \sinh^2 \alpha = N^8. \quad (9.10)$$

This can be achieved by taking α to be large, which translates to the black hole being near extremal. Then we get

$$L(\vec{k}) = 2\pi \sqrt{\vec{k}^2 + N^8 \frac{(k_1 N^3 + k_2 N^2 + k_3 N + k_4)^2}{1 + N^2 + N^4 + N^6}}. \quad (9.11)$$

We now introduce new integers m_1, m_2, m_3, m_4 via

$$k_4 = m_4 - N m_3, \quad k_3 = m_3 - N m_2, \quad k_2 = m_2 - N m_1, \quad k_1 = m_1, \quad (9.12)$$

or equivalently,

$$m_1 = k_1, \quad m_2 = k_2 + Nk_1, \quad m_3 = k_3 + Nk_2 + N^2k_1, \quad m_4 = k_4 + Nk_3 + N^2k_2 + N^3k_1, \quad (9.13)$$

so that

$$L(\vec{k}) = 2\pi \sqrt{N^2(m_1^2 + m_2^2 + m_3^2 + m_4^2 + a_{ij}m_im_j) + k_5^2 + k_6^2}, \quad a_{ij} = \mathcal{O}(N^{-1}). \quad (9.14)$$

The transformation (9.12) is unimodular, so any cycle on T^6 will correspond to some choice of integers $m_1, m_2, m_3, m_4, k_5, k_6$. (9.14) now shows that T^6 can be considered as product of six circles, four each of radius N and two each of radius 1. This produces a near horizon geometry where

$$\tilde{R}_H \sim N, \quad R_H \sim 1, \quad g_H^{(4)} \sim N^{-2}. \quad (9.15)$$

We see from (9.6) that this would give $g_B \sim 1$, $R_B \sim 1$ and $\tilde{R}_B \sim N$. Therefore we have not yet established the existence of a region where R_B is large. This can be rectified by making a continuous $\text{SL}(2, \mathbb{R})$ duality transformation of the solution.¹¹ This leaves the T^6 moduli unchanged, and changes the axion dilaton moduli τ whose imaginary part is $(g_H^{(4)})^{-2}$. Since we also want to preserve the asymptotic value of τ , we are only allowed an $SO(2)$ subgroup of the $\text{SL}(2, \mathbb{R})$ group. If for definiteness we take the asymptotic value of τ to be i then the allowed transformation is $\tau \rightarrow (\tau \cos \theta + \sin \theta) / (-\tau \sin \theta + \cos \theta)$. So if we denote the near horizon value of τ corresponding to (9.15) by $i\Lambda$ for some large Λ , then after the duality transformation we get

$$\tau = \frac{i\Lambda \cos \theta + \sin \theta}{-i\Lambda \sin \theta + \cos \theta} = i \frac{\Lambda}{\Lambda^2 \sin^2 \theta + \cos^2 \theta} + \frac{(1 - \Lambda) \cos \theta \sin \theta}{\Lambda^2 \sin^2 \theta + \cos^2 \theta}. \quad (9.16)$$

If we take $\sin \theta \sim \Lambda^{-\eta}$ with $\frac{1}{2} < \eta < 1$, then we get

$$g_H^{(4)} = \tau_2^{-1/2} \sim \Lambda^{\frac{1}{2} - \eta} \quad (9.17)$$

Using the fact that $\Lambda \sim N^4$ in this case, we get the configuration

$$\tilde{R}_H \sim N, \quad R_H \sim 1, \quad g_H^{(4)} \sim N^{2-4\eta}. \quad (9.18)$$

(9.6) now gives

$$g_B \sim 1, \quad \tilde{R}_B \sim N^{2\eta-1}, \quad R_B \sim N^{2-2\eta}. \quad (9.19)$$

¹¹A continuous duality transformation acting on an electrically charged black hole produces black holes carrying both electric and magnetic charges. Generically these charges do not satisfy quantization law, but for large charges we can simply pick the nearest allowed values of the charges after duality rotation without significantly affecting the near horizon geometry.

Hence for $\frac{1}{2} < \eta < 1$, both R_B and $\tilde{R}_B$ are large, and g_B remains finite.

This shows that we can find a black hole solution carrying appropriate electric and magnetic charges for which in the region (9.10) space-time is best described as an F-theory background with large size of the base $B = T^2/\mathbb{Z}_2$ and hence the (s, r) seven branes present in the F-theory compactification will get exposed in this region. Furthermore, as discussed in [11–13], by scaling the parameter m and the coordinate ρ by a large number we can make the region in the 3+1 dimensional space-time, where the F-theory description is valid, to have arbitrarily large size.

9.2 End of the world E_8 branes from M-theory compactification

We consider now $E_8 \times E_8$ heterotic string theory on T^6 with unbroken $E_8 \times E_8$. This has a dual description as M-theory compactified on $S^1/\mathbb{Z}_2 \times T^6$. $S^1/\mathbb{Z}_2$ can be regarded as the segment of a real line and we have end of the world 9-branes at the two ends of the line segment where the E_8 gauge theories live [21]. Our goal will be to explore what kind of field configurations in the heterotic string theory on T^6 will produce large space-time regions where the M-theory description is valid and the end of the world E_8 branes get exposed to an asymptotic observer..

Let us denote by R_M the size of the interval and by $\tilde{R}_M$ the size of the circles of T^6 , measured in eleven dimensional Planck scale. For the eleven dimensional M-theory description to be valid, we need R_M and $\tilde{R}_M$ to be large. Now, this theory can also be regarded as heterotic string theory on T^6 , with the ten dimensional heterotic string coupling g_H , the size $\tilde{R}$ of T^6 measured in the heterotic string metric and the four dimensional heterotic string metric $g_H^{(4)}$ given by [21],

$$g_H = R_M^{3/2}, \quad \tilde{R}_H = R_M^{1/2} \tilde{R}_M, \quad g_H^{(4)} = g_H \tilde{R}_H^{-3} = \tilde{R}_M^{-3}. \quad (9.20)$$

Our goal is to have R_M and $\tilde{R}_M$ large. To achieve this, we modify the solution described in section 9.1 by taking,

$$\vec{n} = (1, N^{-1}, N^{-2}, N^{-3}, N^{-4}, N^{-5}) / \sqrt{1 + N^{-2} + N^{-4} + N^{-6} + N^{-8} + N^{-10}}, \quad (9.21)$$

for some large integer N . Then using (9.7) we see that a cycle in T^4 that joins the point $\vec{0}$ to $2\pi\vec{k}$ in the $(x^4, x^5, x^6, x^7, x^8, x^9)$ directions for some integers $k_1, k_2, k_3, k_4, k_5, k_6$, has length

$$L(\vec{k}) = 2\pi \sqrt{\vec{k}^2 + \frac{2m}{\rho} \sinh^2 \alpha (\vec{k} \cdot \vec{n})^2} \quad (9.22)$$

So for the choice

$$\frac{2m}{\rho} \sinh^2 \alpha = N^{12}, \quad (9.23)$$

we get

$$L(\vec{k}) = 2\pi \sqrt{\vec{k}^2 + N^{12} \frac{(k_1 N^5 + k_2 N^4 + k_3 N^3 + k_4 N^2 + k_5 N + k_6)^2}{1 + N^2 + N^4 + N^6 + N^8 + N^{10}}}. \quad (9.24)$$

If introduce new integers $m_1, m_2, m_3, m_4, m_5, m_6$ via

$$k_1 = m_1, \quad k_i = m_i - N m_{i-1}, \quad \text{for } 2 \leq i \leq 6, \quad (9.25)$$

then

$$L(\vec{k}) = 2\pi N \sqrt{m_1^2 + m_2^2 + m_3^2 + m_4^2 + m_5^2 + m_6^2 + a_{ij} m_i m_j}, \quad a_{ij} = \mathcal{O}(N^{-1}). \quad (9.26)$$

(9.26) now shows that for large N , T^6 can be considered as product of six circles, each of radius N . This gives $\tilde{R}_H \sim N$. Also for this background, (9.7) gives $g_H^{(4)} \sim N^{-3}$. However by making a continuous $\text{SL}(2, \mathbb{R})$ duality transformation of the type described at the end of section 9.1, we can make $g_H^{(4)} \sim N^{3-6\eta}$ for $\frac{1}{2} < \eta < 1$. This gives

$$\tilde{R}_H \sim N, \quad g_H^{(4)} \sim N^{3-6\eta}. \quad (9.27)$$

(9.20) now gives

$$\tilde{R}_M \sim N^{2\eta-1}, \quad R_M \sim N^{4(1-\eta)}. \quad (9.28)$$

Hence for $\frac{1}{2} < \eta < 1$ we shall achieve large values of R_M and $\tilde{R}_M$.

This shows that an observer in the heterotic string theory on $T^6 \times \mathbb{R}^{3,1}$ can study M-theory on an interval and the end of the world E_8 branes to any degree of accuracy.

9.3 D8-branes in type I' string theory

D8-branes are codimension one branes in ten dimensional type IIA string theory, but instead of having the usual Minkowski vacua of type IIA string theory on two sides, it separates AdS vacua of type IIA string theory known as the Romans supergravity [23]. While we do not have direct world-sheet construction of a string theory whose low energy limit is given by Romans supergravity, the latter appears as part of the background in type IIA string theory compactified on $S^1/\mathbb{Z}_2$, with orientifold 8-planes at the two ends and 16 D8-branes placed at various points along $S^1/\mathbb{Z}_2$ [22]. This description becomes more and more accurate in the limit when the size R_A of the interval measured in the type IIA metric grows but the string coupling

remains finite or small.¹² We shall call the $S^1/\mathbb{Z}_2$ direction as the 9th direction. We shall consider four dimensional string theory by compactifying this further on a five dimensional torus of size of order $\tilde{R}_A$ which we also take to be large. We shall call these additional compact directions as 4,5,6,7,8 directions. Our goal will be to first map this to a heterotic string compactification on T^6 via a series of duality transformation and then identify a black hole solution whose near horizon geometry produces large R_A and $\tilde{R}_A$ with small but finite g_A .

We first make a T-duality along the 9-th direction to convert it to a type I theory, with the type I string coupling g_I and the size R_I of the 9-th direction and size $\tilde{R}_I$ of the 4,5,6,7,8 directions, measured in the type I metric, given by

$$R_I \sim R_A^{-1}, \quad g_I \sim g_A R_A^{-1}, \quad \tilde{R}_I \sim \tilde{R}_A. \quad (9.29)$$

In Planck units, the sizes r of the 9th direction and $\tilde{r}$ of the 4,5,6,7,8 directions are:

$$\tilde{r} \sim \tilde{R}_I g_I^{-1/4} \sim \tilde{R}_A R_A^{1/4} g_A^{-1/4}, \quad r \sim R_I g_I^{-1/4} \sim R_A^{-3/4} g_A^{-1/4}. \quad (9.30)$$

We now go to the heterotic description using the heterotic type I duality. The heterotic coupling g_H is given by $1/g_I$ up to a constant of proportionality and the radii of the circles measured in Plank units remain unchanged. Denoting by R_H and $\tilde{R}_H$ the sizes of the 9-th direction and the 4,5,6,7,8 directions measured in the heterotic string metric, we get

$$g_H \sim 1/g_I \sim g_A^{-1} R_A, \quad \tilde{R}_H \sim \tilde{r} g_H^{1/4} \sim \tilde{R}_A R_A^{1/2} g_A^{-1/2}, \quad R_H \sim r g_H^{1/4} \sim R_A^{-1/2} g_A^{-1/2}. \quad (9.31)$$

From this we get the four dimensional heterotic string coupling to be

$$g_H^{(4)} \sim g_H \tilde{R}_H^{-5/2} R_H^{-1/2} \sim g_A^{1/2} \tilde{R}_A^{-5/2}. \quad (9.32)$$

We see from (9.31) that in the limit of large R_A , $\tilde{R}_A$ with fixed g_A , $\tilde{R}_H$ becomes large but R_H becomes small. To bring this to a form analyzed earlier where all circles have large size in the heterotic description, we make a further T-duality transformation along the 9-direction sending R_H to R_H^{-1} but leaving $\tilde{R}_H$ and $g_H^{(4)}$ unchanged. This maps the $SO(32)$ heterotic string

¹²As in the case of F-theory, the type IIA string coupling varies over the interval $S^1/\mathbb{Z}_2$ unless each of the orientifold 8-plane has 8 D8-branes on it [22]. When each orientifold plane has 8 D8-branes near it, then the string coupling remains constant away from the orientifold D8-brane system and the g_A appearing in the various formulae below can be taken to be the value of the string coupling in this region. In the limit where the D8-branes are on top of the orientifold planes, the $SO(32)$ group is broken to $SO(16) \times SO(16)$. This is further broken to $U(1)^{16}$ when the D8-branes are pulled away from the orientifold planes and from each other.

theory to $E_8 \times E_8$ heterotic string theory, both broken (approximately) to $SO(16) \times SO(16)$ by a Wilson line along the 9-th direction. Denoting the resulting parameters by $'$ we get

$$\tilde{R}'_H \sim \tilde{R}_A R_A^{1/2} g_A^{-1/2}, \quad R'_H \sim R_A^{1/2} g_A^{1/2}, \quad g_H^{(4)'} \sim g_A^{1/2} \tilde{R}_A^{-5/2}. \quad (9.33)$$

Finally, inverting (9.33) we get

$$g_A \sim \left(g_H^{(4)'}\right)^{-1/2} \tilde{R}'_H^{-5/4} R_H'^{5/4}, \quad \tilde{R}_A \sim \left(g_H^{(4)'}\right)^{-1/2} \tilde{R}'_H^{-1/4} R_H'^{1/4}, \quad R_A \sim \left(g_H^{(4)'}\right)^{1/2} \tilde{R}'_H^{5/4} R_H'^{3/4}. \quad (9.34)$$

Our goal will be to produce the configuration (9.33) with large R_A and $\tilde{R}_A$ but finite g_A near the horizon of an appropriate black hole. (9.33) shows that in order to get large R_A and $\tilde{R}_A$ with fixed g_A we need both $\tilde{R}'_H$ and R'_H to be large but $\tilde{R}'_H$ should be large compared to R'_H and $g_H^{(4)'}$ should be small. To achieve this we consider the solution considered in section 9.2 but replace (9.23) by

$$\frac{2m}{\rho} \sinh^2 \alpha = N^{12-\gamma}, \quad 0 < \gamma < 2. \quad (9.35)$$

In this case (9.26) is replaced by

$$L(\vec{k}) = 2\pi N \sqrt{m_1^2 + m_2^2 + m_3^2 + m_4^2 + m_5^2 + N^{-\gamma} m_6^2 + a_{ij} m_i m_j}, \quad a_{ij} = \begin{cases} \mathcal{O}(N^{-1}) & \text{for } i \neq j \\ \mathcal{O}(N^{-2}) & \text{for } i = j \end{cases}. \quad (9.36)$$

This gives

$$\tilde{R}'_H \sim N, \quad R'_H \sim N^{1-\gamma/2}. \quad (9.37)$$

Also we have $g_H^{(4)'} \sim N^{-3+\frac{1}{4}\gamma}$, but with the help of a continuous duality rotation, we can convert it to

$$g_H^{(4)'} \sim N^{(-3+\frac{1}{4}\gamma)(2\eta-1)}, \quad (9.38)$$

as before. Requiring that g_A computed from (9.33) is of order unity, now gives

$$2\eta - 1 = \frac{5\gamma}{12 - \gamma}. \quad (9.39)$$

Substituting this in (9.36), (9.38) and using (9.33), we now get

$$R_A = N^{2-\gamma}, \quad \tilde{R}_A \sim N^{\gamma/2}. \quad (9.40)$$

Therefore, for $0 < \gamma < 2$, we have large R_A and $\tilde{R}_A$ as desired.

This shows that an observer in heterotic string theory on $T^6 \times \mathbb{R}^{3,1}$ can not only study D8-branes but also Romans supergravity in $\mathbb{R}^{9,1}$ to any desired degree of accuracy. There is however a limit on the cosmological constant of the Romans supergravity that can be studied this way since we only have a limited number of D8-branes.

10 Flat codimension one branes

We have seen that codimension one branes are difficult to study since macroscopic loop of such branes are hidden behind the event horizon. In special cases when they are part of compactification, one may be able to study them by producing a large region in space-time where the compact directions tangential and transverse to the brane become large, exposing the brane to an asymptotic observer. Examples of this for BPS branes were discussed in section 9. However this generally does not work for non-supersymmetric branes, *e.g.* there is no known compactification in which the conjectured 8-brane separating type IIA and type IIB vacua appears in the vacuum.

In this section we shall explore if flat codimension one branes can be studied by an asymptotic observer. While flat codimension one branes have infinite energy and hence cannot be regarded as a state of the theory, we could imagine a scenario where they are present as part of the background, breaking translation invariance along directions transverse to the brane. We shall call the two sides of the brane as A and B and also label the vacua on the two sides as A and B . The question that we shall be addressing is the following. Can an observer residing on side A of the brane send an apparatus to the side B which performs an experiment over a macroscopic time scale and then sends the information back to the original observer residing on side A ? If this is the case then we can say that the observer at infinity can measure properties of the vacua on both sides of the brane to arbitrary accuracy, and the brane achieves the purpose of unifying the vacua on two sides.

For this analysis we need to postulate some of the properties of the brane. If the brane sources any other scalar field ϕ then away from the brane we shall have a background in which ϕ depends on the coordinate z transverse to the brane. This will produce non-zero energy momentum tensor and the geometry away from the brane will not be vacuum of the theories A and B as desired. To avoid this situation we shall assume that the brane acts as a source of only the gravitational field and no other massless field.¹³ Put another way, we assume that sufficiently far away from the brane all fields other than the metric become constant and the tension of the brane is measured by integrating the energy momentum tensor to sufficient distance in the transverse direction where variation of the other fields have died down. We shall further assume that the component of the energy-momentum tensor along the brane is proportional to the induced metric on the brane and the component of the energy momentum

¹³This would hold automatically if the brane separates two Minkowski vacua in each of which all moduli have been stabilized.

tensor transverse to the brane vanishes.

The metric produced by such a brane was found in [42]. For a $(D - 1)$ -brane in a $(D + 1)$ dimensional theory, it takes the form:

$$ds^2 = e^{-2|\sigma|/\kappa} \left[-d\tau^2 + d\sigma^2 + e^{2\tau/\kappa} \sum_{i=1}^{D-1} (dy^i)^2 \right]. \quad (10.1)$$

Here τ is the time coordinate, σ is the coordinate transverse to the brane, y^i are the coordinates tangential to the brane and κ is a constant related to the brane tension. It was also shown in [42] that away from the location of the brane the space-time metric is flat. Hence we can go to a coordinate system in which the metric away from the brane takes the form of the Minkowski metric. The coordinate transformation is given by

$$t = \kappa e^{|\sigma|/\kappa} \sinh \frac{\tau}{\kappa} + \frac{1}{2\kappa} e^{\tau/\kappa} \vec{y}^2, \quad z = \kappa e^{|\sigma|/\kappa} \cosh \frac{\tau}{\kappa} - \frac{1}{2\kappa} e^{\tau/\kappa} \vec{y}^2, \quad x^i = e^{(\tau+|\sigma|)/\kappa} y^i. \quad (10.2)$$

The metric in this new coordinate system takes the form:

$$ds^2 = -dt^2 + dz^2 + \sum_{i=1}^{D-1} dx^i dx^i. \quad (10.3)$$

However in this coordinate system the brane moves along the transverse direction with uniform acceleration, with the acceleration determined by the tension of the brane. We shall denote by t the time and by z the coordinate transverse to the brane in this frame. Then the trajectory of the $\vec{y} = 0$ point on the brane in the z - t plane takes the form

$$t = \kappa \sinh \frac{\tau}{\kappa}, \quad z = \kappa \cosh \frac{\tau}{\kappa}, \quad (10.4)$$

with the acceleration of the brane given by κ^{-1} . Physically this is a reflection of the fact that the brane exerts a constant repulsive gravitational force that makes an object outside accelerate away from the brane. So in the inertial frame of the object the brane will accelerate away from the object.

Let us now examine the task at hand. Let the original observer be on side A of the brane. If it is within the past light cone of the brane, it can send an apparatus across the brane to the other side B . After reaching side B , the apparatus can undergo successive motion of acceleration and free fall any number of times and eventually send back the information gathered to the observer on the side A . The actual experiment has to be done during the free fall part of the trajectory since this is the inertial frame in which side B will appear to be in

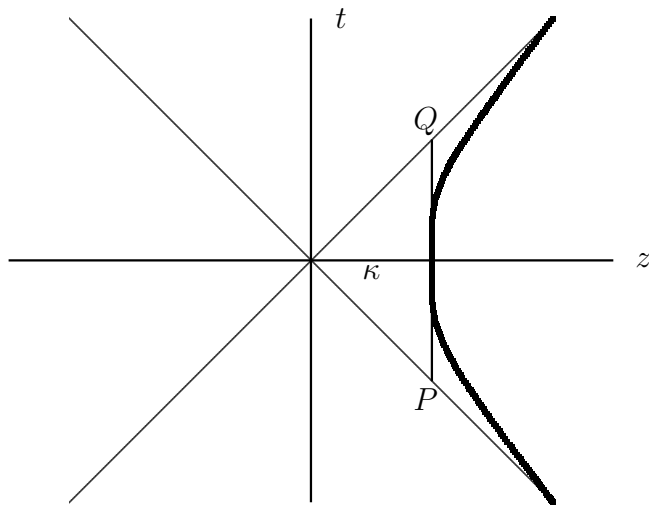


Figure 2: This figure illustrates the maximum period during which an apparatus coming from side A of the brane to side B and then returning to A can be in the inertial frame. The thick curve is the trajectory of the brane as viewed from side B , which is on the left side of the thick curve. The line segment PQ denotes the part of the trajectory of the apparatus on side B along which the apparatus is in free fall. The view from side A has not been shown in this figure but it is of the same form as from side B .

its vacuum. Thus the question we want to address is: can we make a free fall phase last for arbitrarily long time, measured by the clock in the apparatus?

To answer this question we can assume, without loss of generality, that the free fall phase in which the proper time of the apparatus is maximum corresponds to constant z coordinate. This can be achieved by going to a boosted frame in which the trajectory (10.4) remains invariant but the inertial apparatus is at rest. In that case the proper time measured by the inertial observer will be given by the lapse of t along the trajectory. The maximum possible time interval for which such an observer can exist is seen to be 2κ according to Fig. 2. Since κ is fixed by the tension of the brane, we see that this time cannot be made arbitrarily large. Therefore we conclude that the existence of such a brane does not allow an observer on side A to make measurement on side B to arbitrary accuracy.

We end by noting that if the observer itself is allowed to cross the brane and go to the other side, then it is possible for the world-line of the observer to have infinitely large inertial segments on both sides of the brane. This can be seen from Fig. 2 by regarding this as representing the side A instead of side B . There are clearly infinitely long time-like geodesics (e.g. vertical lines) that intersect the trajectory of the brane. Such a trajectory spends infinitely long period in A

and then crosses the brane. Once it emerges on the side B , the allowed trajectories can again be studied from Fig. 2, this time regarding this as the side B . Without loss of generality we can take the point of emergence as the point where the trajectory intersects the real axis and we see that a vertical trajectory emerging from this point can exist for infinite time as long as it does not intend to return to A . Such an observer will be able to make arbitrarily accurate measurements on both sides. This will be in accordance with the recent proposal of tying the measurements to the world-line of a single observer instead of to the asymptotic region [43,44]. This however postulates the existence of a meta observer who can exist on both sides of the brane, even though the elementary particles themselves differ on the two sides. This should be distinguished from the case where one needs to only send an apparatus to the other side, which may be achieved *e.g.* by an appropriate gravitational wave signal that constructs the apparatus after crossing to the other side, performs the desired experiment and sends back the result via gravitational wave signal.

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